



# Mixed Strategies

- It would be a pretty bad idea to play any deterministic strategy in matching pennies

|       | Heads | Tails |
|-------|-------|-------|
| Heads | 1, -1 | -1, 1 |
| Tails | -1, 1 | 1, -1 |



# Mixed Strategies



- It would be a pretty bad idea to play any deterministic strategy in matching pennies
- Idea: confuse the opponent by playing **randomly**
- Define a **strategy**  $s_i$  for agent  $i$  as any probability distribution over the actions  $A_i$ .
  - **pure strategy**: only one action is played with positive probability
  - **mixed strategy**: more than one action is played with positive probability
    - these actions are called the **support** of the mixed strategy
- Let the set of **all strategies** for  $i$  be  $S_i$
- Let the set of **all strategy profiles** be  $S = S_1 \times \dots \times S_n$ .

# Utility under Mixed Strategies

- What is your **payoff** if all the players follow mixed strategy profile  $s \in S$ ?
  - We can't just read this number from the game matrix anymore: we won't always end up in the same cell



# Utility under Mixed Strategies



- What is your **payoff** if all the players follow mixed strategy profile  $s \in S$ ?
  - We can't just read this number from the game matrix anymore: we won't always end up in the same cell
- Instead, use the idea of **expected utility** from decision theory:

$$u_i(s) = \sum_{a \in A} u_i(a) Pr(a|s)$$

$$Pr(a|s) = \prod_{j \in N} s_j(a_j)$$

# Best Response and Nash Equilibrium

Our definitions of best response and Nash equilibrium generalize from actions to strategies.

## Definition (Best response)

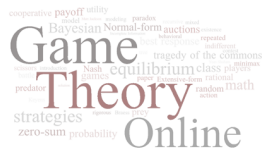
$$s_i^* \in BR(s_{-i}) \text{ iff } \forall s_i \in S_i, u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i})$$

## Definition (Nash equilibrium)

$$s = \langle s_1, \dots, s_n \rangle \text{ is a Nash equilibrium iff } \forall i, s_i \in BR(s_{-i})$$



# Best Response and Nash Equilibrium



Our definitions of best response and Nash equilibrium generalize from actions to strategies.

## Definition (Best response)

$s_i^* \in BR(s_{-i})$  iff  $\forall s_i \in S_i, u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i})$

## Definition (Nash equilibrium)

$s = \langle s_1, \dots, s_n \rangle$  is a **Nash equilibrium** iff  $\forall i, s_i \in BR(s_{-i})$

## Theorem (Nash, 1950)

*Every finite game has a Nash equilibrium.*





# Example: Coordination



|       | Left | Right |
|-------|------|-------|
| Left  | 1, 1 | 0, 0  |
| Right | 0, 0 | 1, 1  |

# Example: Prisoner's Dilemma



|     | $C$      | $D$      |
|-----|----------|----------|
| $C$ | $-1, -1$ | $-4, 0$  |
| $D$ | $0, -4$  | $-3, -3$ |